



Predicting atmospheric delays for rapid ambiguity resolution in precise point positioning

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Abstract

Integer ambiguity resolution in precise point positioning (PPP) can shorten the initialization and re-initialization time, and ambiguity-fixed PPP solutions are also more reliable and accurate than ambiguity-float PPP solutions. However, signal interruptions are unavoidable in practical applications, particularly while operating in urban areas. Such signal interruptions can cause discontinuity of carrier phase arc, which introduces new integer ambiguities. Usually it will take approximately 15 min of continuous tracking to a reasonable number of satellites to fix new integer ambiguities. In many applications, it is impractical for a PPP user to wait for such a long time for the re-initialization. In this paper, a method for rapid ambiguity fixing in PPP is developed to avoid such a long re-initialization time. Firstly, the atmospheric delays were estimated epoch by epoch from ambiguity-fixed PPP solutions before the data gap or cycle slip occurs. A random walk procedure is then applied to predict the atmospheric delays accurately over a short time span. The predicted atmospheric delays then can be used to correct the observations which suffer from signal interruptions. Finally, the new ambiguities can be fixed with a distinct WL-LX-L3 (here LX denotes either of L1, L2) cascade ambiguity resolution strategy. Comprehensive experiments have demonstrated that the proposed method and strategy can fix zero-difference integer ambiguities successfully with only a single-epoch observation immediately after a short data gap. This technique works even when all satellites are interrupted at the same time. The duration of data gap bridged by this technique could be possibly extended if a more precise atmospheric delay prediction is found or on-the-fly (OTF) technology is applied. Based on the proposed method, real-time PPP with integer ambiguity fixing becomes more feasible in practice.

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1. Introduction

Precise Point Positioning (PPP) technique (Zumberge et al., 1997) is a focus of the international GNSS community, as it can provide decimeter to centimeter level positioning accuracy globally with no requirement of a dense network of reference stations. It has been widely applied in precise orbit determination, geodesy, aerial photogrammetry, glaciology, GPS meteorology, precise timing, etc. (Kouba and Héroux, 2001; Gao and Shen, 2001; Bisnath and Gao, 2008). However, traditional PPP with float

ambiguities requires quite a long convergence time of about 20 min to achieve centimeter-level positioning accuracy, and its reliability is lower than an ambiguity-fixed baseline solution (Han and Rizos, 1996; Wang et al., 2002, 1998a,b). It cannot satisfy some timely applications such as real-time kinematic positioning (Zhang et al., 2011a,b). In order to shorten the initialization time and improve the accuracy and reliability of PPP, some methods for integer-ambiguity-fixed PPP solutions have been proposed in recent years (Ge et al., 2008; Laurichesse et al., 2008; Collins et al., 2008; Bertiger et al., 2010; Li and Zhang, 2012). The reported results showed that initialization and re-initialization time can be shortened to about 15 min by applying the LAMBDA (Teunissen, 1995) method to integer ambiguity resolution in PPP. Positioning

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accuracy at centimeter level can be obtained once the integer ambiguities are fixed.

Although ambiguity fixing can shorten the convergence time of PPP to some extent, in practical applications, especially in an urban area, signal blockage or interruption will result in frequent integer ambiguity resets. Such a long time for fixing new integer ambiguities is not acceptable for most users. If one could fix the zero-difference (ZD) integer ambiguities within seconds, it would be of great benefit for real-time kinematic positioning. Let us recall that in Network Real-Time Kinematic (NRTK) positioning, as most error sources can be eliminated, the integer ambiguities can be fixed and validated very shortly; see, for instance, Section 6.1 and Remark 6.3 in Lannes and Prieur (2013). However, in PPP, clock biases, orbit, and atmosphere delay are significant limitations for rapid ambiguity resolution. In recent years, the quality of orbit and clock from International GNSS Service (IGS) has improved significantly (Dow et al., 2009), the radial accuracy of ultra-rapid orbit is about 5 cm (<http://igsb.jpl.nasa.gov/>). Furthermore, the high correlation between radial orbit component and satellites clocks allows the geometrical errors caused by the orbits to be compensated by clock estimation. Although the predicted satellite clock accuracy of IGS is only at 3 ns level, the real-time estimated and short-term predicted clock accuracy is up to about 0.1 ns, which meets the demands of rapid ambiguity resolution in real-time PPP (Ge et al., 2012; Li et al., 2013). The remaining ionospheric delay should be considered carefully due to the irregularity of its temporal and spatial variation (Deng et al., 2009).

Several PPP-RTK methods have been developed to improve the performance of global PPP service in specific areas by making use of available regional reference networks (Wübbena et al. 2005; Teunissen et al. 2010; Li et al. 2011; Zhang et al. 2011a,b). Let us note in passing that instantaneous ambiguity resolution does not raise any technical problem. A PPP-RTK theoretical framework is also given by Lannes and Teunissen (2011) and Lannes and Prieur (2013). As stressed in those last papers, the variance-covariance matrix of the satellite-clock biases should be used for implementing the PPP mode properly. To the best of our knowledge, this is not done in practice. Concerning the ionospheric delay, its value can be estimated on the grounds of observations at regional reference stations. However, in global PPP ambiguity resolution (PPP-AR), precise atmospheric delay models cannot be derived from the sparse global reference network. Recently, some cycle slip fixing (Banville and Langley, 2009; Zhang and Li, 2012) and rapid re-convergence (Geng 2009; Geng et al., 2010; Li et al., 2013) methods have been proposed to improve the PPP performance. In this contribution, we present a way of estimating accurate ionospheric delays from the ZD ambiguity-fixed PPP solutions in previous epochs and then predict them in a correct way to fill the data gaps. The time-smoothness of the ionosphere (Dai et al., 2003; Teunissen and Bakker, 2012) is exploited for

use in rapid ambiguity resolution in global PPP. The performance of the proposed method is demonstrated in different observational environments.

2. Ionospheric delay and its prediction

Reducing or eliminating atmospheric delays is important for ambiguity resolution. These delays have to be kept as minimum as possible in order to resolve ambiguities reliably. Ignoring satellite orbit and clock errors, the simplified phase observation equation can be written as follows (Li and Zhang, 2012):

$$L_i^k = \rho_{ig}^k - I_i^k + T_i^k + f_i + f^k + \lambda N_i^k + \varepsilon_i^k \quad (1)$$

where, the superscript k refers to a given satellite, and subscript i refers to a given receiver; L_i^k could be original phase observations or a combination of phase observations, such as L1, L2, or wide-lane (WL) combination, ρ_{ig}^k denotes geometric distance, I_i^k is the ionospheric delay, T_i^k is the tropospheric delay, f_i is the uncalibrated fractional offsets (UFOs) related to receiver, f^k is the UFOs related to satellite, N is the ZD integer ambiguity, ε is the observation noise. Other error components such as the phase center offsets and variations, phase wind-up, relativistic effect, tide loading and so on could be precisely corrected with existing models.

Due to the existence of UFOs originating at receiver and satellite, for a long time only double-differenced ambiguities between satellites and receivers can be fixed. In the recent years, it was demonstrated that satellite UFOs could be estimated from a reference network and applied to other stations for fixing integer ambiguity in PPP mode (Ge et al. 2008; Collins et al., 2008; Laurichesse et al. 2008). Thus, PPP with integer ambiguity fixing requires not only precise satellite orbit and high-rate satellite clock corrections but also UFOs product. Since tropospheric delay can be estimated precisely by introducing zenith path delay (ZPD) parameter in PPP, ionospheric delay is the remaining bias that should be considered carefully (Jin et al., 2010; Jin et al., 2012). In the following sections, we will focus on developing a way to estimate accurate ionospheric delay from epochs where the ambiguities have been fixed and then predict it in a correct manner to fill the data gap.

2.1. Epoch-by-epoch ionospheric delay estimation

Provided that (at the end of the initialization stage) the ZD integer ambiguities have been successfully fixed, coordinates and ZPD with cm (even mm) level accuracy can be obtained with the GPS observations collected during the initialization stage at the PPP user end. That means all parameters are accurately known in Eq. (1) except I_i^k . Thus, it is straightforward to compute zero-difference ionospheric delay accurately with the following equation:

$$I_i^k = \rho_{ig}^k - L_i^k + T_i^k + f_i + f^k + \lambda N_i^k + \varepsilon_i^k \quad (2)$$

The accuracy of the estimated ZD ionospheric delay depends on the quality of estimated UFOs. Generally speaking, the quality of UFOs estimated from a reference network should be high, since observations from reference stations are with considerable redundancy and high quality. The NL UFOs are primarily estimated by using carrier phases, its accuracy can be up to one tenth of the NL wavelength. WL UFOs are derived from both pseudo-ranges and carrier phases using the Melbourne-Wübbena combination (Wübbena, 1985), its accuracy could be about one tenth of the WL wavelength. This is sufficient for WL ambiguity resolution, but the error of WL UFOs will cause a bias in the ionospheric delay estimation. Fortunately, it will have no effect on the subsequent ionosphere prediction and positioning.

Assuming UFOs of receiver and satellite are biased with $Bias_i$, $Bias^k$ respectively, \hat{f}_i^k denotes the sum of estimated satellite and receiver UFOs, f_i^k denotes the truth value. We have:

$$\hat{f}_i^k = f_i^k + Bias_i + Bias^k \quad (3)$$

Let I_i^k be true value of the ionospheric delay, \hat{I}_i^k be the estimated ionospheric delay biased by UFOs error. We have,

$$\hat{I}_i^k = I_i^k + Bias_i + Bias^k \quad (4)$$

As UFOs are very stable over a short period of time (e.g. several minutes), $Bias_i$ and $Bias^k$ can be considered as time-constant for a short period of time to allow temporal ionospheric modeling. At the user end, the observation Eq. (1) is used, and the $Bias^k$ in predicted ionospheric delay will be compensated by $Bias^k$ in UFOs due to their equal size and opposite sign in Eq. (1). Meanwhile, the effect of the $Bias_i$ part on the estimated ionosphere correction is the same to all visible satellites at the user end. It can be absorbed by receiver clock parameter. Therefore, such a systematic bias in estimated ionospheric delays caused by WL UFOs error has no influence on ambiguity resolution (AR) and positioning processing.

2.2. Temporal prediction of ionospheric delay

In the previous section, we calculated precise ionospheric delay epoch by epoch from ZD observations. The tropospheric delay can also be estimated accurately in PPP. Therefore, the atmospheric delays at each epoch are available after initialization. Dai et al. (2003) have done numerous studies on temporal correlation of atmospheric errors, and proposed feasible methods to model and predict atmospheric delays with the random walk process or linear fitting function. In this paper, a random walk process is adopted for prediction. A weighting strategy has to be employed according to the time latency of the prediction. For a temporal correlation series, its autocorrelation function can be expressed as a function of time interval τ (Gelb, 1979):

$$f(\tau) = \sigma_1^2 \cdot e^{-|\tau|/T} \quad (5)$$

where T stands for the first-order correlation time, σ_1^2 is the variance. Denoting the time interval as τ , the prediction of given quantities is expressed as (Gelb, 1979):

$$I_{t_{k+1}} = e^{-|\tau|/T} \cdot I_{t_k} + w_t, w_t \sim N(0, \sigma_{wt}^2) \quad (6)$$

where the variance of the prediction error w_t is:

$$\sigma^2(\tau)_{wt} = \sigma^2 \cdot (1 - e^{-2|\tau|/T}) \quad (7)$$

The elevation-angle-related weighting function is also taken into account, in which E is the satellite elevation angle:

$$p(E) = \begin{cases} 1, & E \geq 30 \\ 2 \sin(E), & 5 \leq E < 30 \\ 0, & E < 5 \end{cases} \quad (8)$$

3. Rapid ambiguity resolution with the predicted atmospheric delays

As mentioned above, the ZD ionospheric delay and tropospheric delay could be derived from previous fixed epochs after initialization. The atmospheric delays then can be predicted by using a random walk process as mentioned in Section 2.2. The predicted atmospheric corrections then can be applied to correct the ZD carrier phase observations. Immediately after data gaps, the corrected ZD carrier phase observations could be employed to implement the rapid AR. Thus, the re-initialization time can be shortened.

In this section, a WL-LX-L3 (LX represents anyone of L1, L2) cascade ambiguity resolution strategy is applied to obtain a fixed PPP solution. Firstly, WL ambiguity is fixed with the aid of the predicted atmosphere corrections. WL ambiguity is relatively easy to fix as its wavelength is very long (86.3 cm). WL phase observation L_{WL} and WL pseudorange observations P_{WL} are used for WL ambiguity resolution, ionospheric and tropospheric errors are corrected using the predicted atmospheric corrections. Secondly, NL ambiguity could be derived from L3 combination and fixed with the constraint of a WL fixed solution. If the NL ambiguity cannot be fixed, then L1 or L2 will be employed for ambiguity resolution with the aid of predicted atmospheric corrections and WL fixed solution information. Finally, L3 is reconstructed for final positioning as the ionosphere-free combination will not be influenced by residual ionospheric error. The flow chart of this cascade strategy is shown in Fig. 1.

There are several approaches to valid the resolved integer ambiguities, such as R -ratio (Euler and Schaffrin 1991), W -ratio (Wang et al. 1998a) as well as the Integer Aperture-based R -ratio, and W -ratio methods (Verhagen and Teunissen 2006; Li and Wang, 2012). In this study, the well-known R -ratio test was used to validate the ambiguity resolution. The ratio test is generally defined as the ratio of the second minimum quadratic form of the residuals to the minimum quadratic form of residuals. It is used to

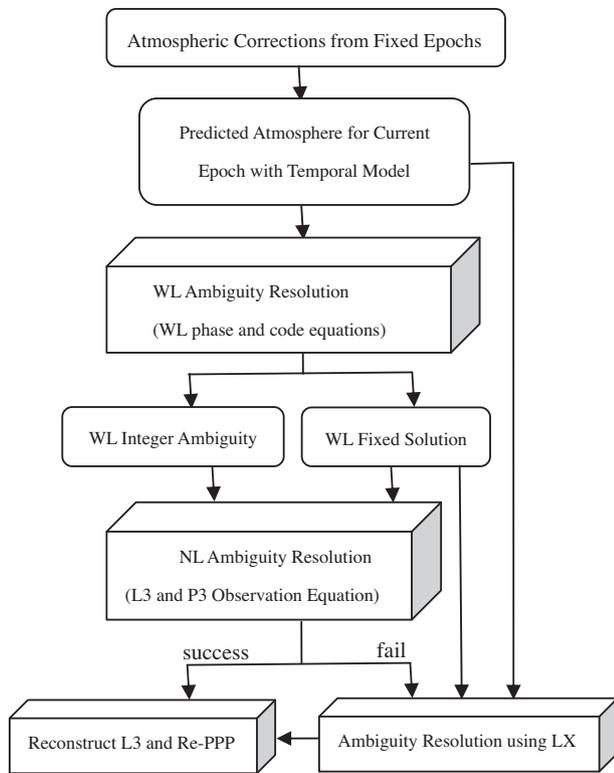


Fig. 1. Flow chart of single-epoch ambiguity resolution.

discriminate between the second set of optimum integer candidates and the optimum one. The critical criterion for the *R*-ratio test is selected as 3 which is generally used in ambiguity validation (Han and Rizos, 1996) (see Table 1).

4. Experiment results and analysis

In this section, we investigate the prediction error of ionospheric delays, and assess this method firstly at static stations and then in kinematic environments.

4.1. Static experiment

In this experiment, ultra-rapid IGS satellite orbit and real-time estimated clock corrections were used. Twelve stations, between which the average distance was about 1500 km, were selected from regional monitoring networks of China to calculate the UFOs for supporting ambiguity resolution in PPP. The 1 Hz GPS data collected at SHAO station (not used for UFOs and clock generation) on

DOY122 in 2008 were used for analyzing the performance of the proposed method.

4.1.1. Epoch-by-epoch ZD ionospheric delays

According to Eq. (2), we can calculate ZD ionospheric delays for all satellites. These ionospheric delays with respect to the corresponding elevation angles for some satellites are typical illustrated in Fig. 2. In this figure, *E* denotes satellite elevation angle and SIN denotes the sine trigonometric function. Apparently, a strong temporal correlation does exist in the ionospheric delays between neighboring epochs. It provides the possibility to simulate its behavior and predict it accurately over a short period of time. These delays also exhibit a high degree of correlation with the elevation angles for the tracked satellites.

4.1.2. Prediction error of ionospheric delay at different latencies

The typical residuals of the predicted ionospheric delays at different latencies (1 s, 60 s, 120 s, 180 s and 300 s) and the elevation angles of four satellites are presented in Fig. 3.

These residuals are the differences between the predicted and estimated ionospheric delays at all epochs, and the estimated ones are considered as a truth against which the accuracy of the predicted ones can be judged. It is obvious that the longer the latency, the bigger the prediction error there will be. The predicted residuals can be accurate up to 5 cm when the latency is within 120 s and over 5 cm for some low elevation-angle satellites when the latency is up to 180 s. With 300 s latency, the residuals of some satellites at very low elevation angle even reached about 1 dm. That means that the lower the elevation angle, the bigger the prediction error. The weighting strategy has to be constructed using both the satellite elevation angle and the time latency of the prediction in the subsequent ambiguity resolution.

4.1.3. Performance of instantaneous AR

We simulated signal interruptions for all the satellites at each epoch, and the single-epoch ambiguity resolution algorithm was employed for positioning epoch by epoch. Fig. 4 gives the positioning errors of single-epoch ambiguity fixed solution using the predicted atmospheric delays with different latencies (1 s, 60 s, 120 s and 180 s). The figures on the left show the results of fixed solution with WL observations at different latencies, the right ones are the

Table 1
Success rate and positioning accuracy of instantaneous ambiguity fixing of WL and L3.

Latency (s)	Success rate of WL (%)	Success rate of L3 (%)	Positioning accuracy of WL (cm)	Positioning accuracy of L3 (cm)
1	100	100	0.2	1.6
60	99.7	99.7	2.1	1.7
120	99.2	99.2	3.2	1.7
180	96.4	96.4	4.5	1.8

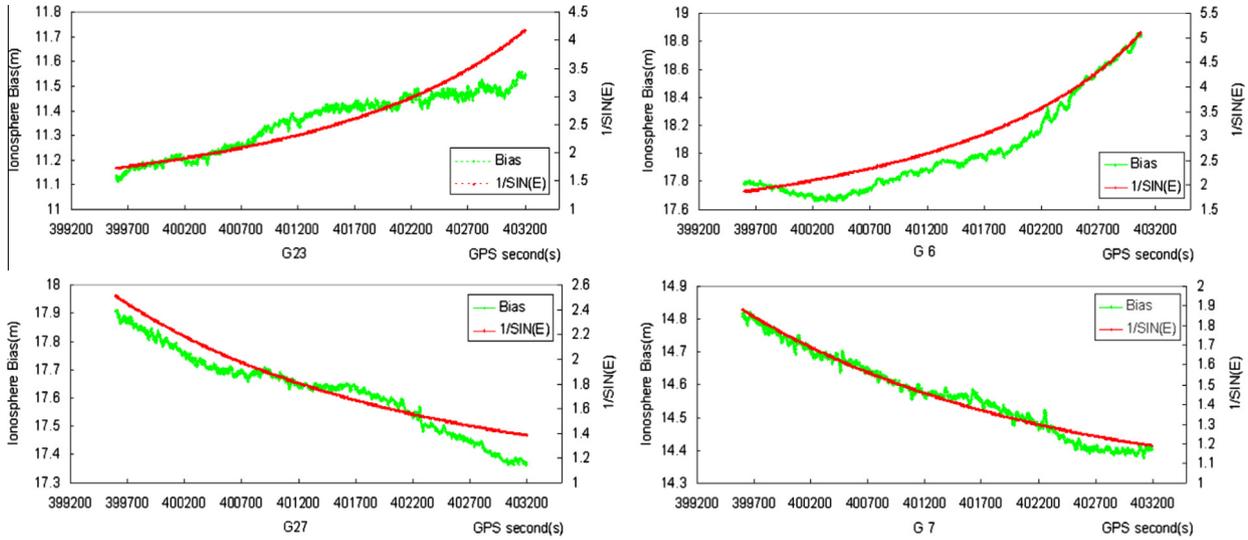


Fig. 2. Estimated ZD ionospheric delay and corresponding $1/SIN(E)$.

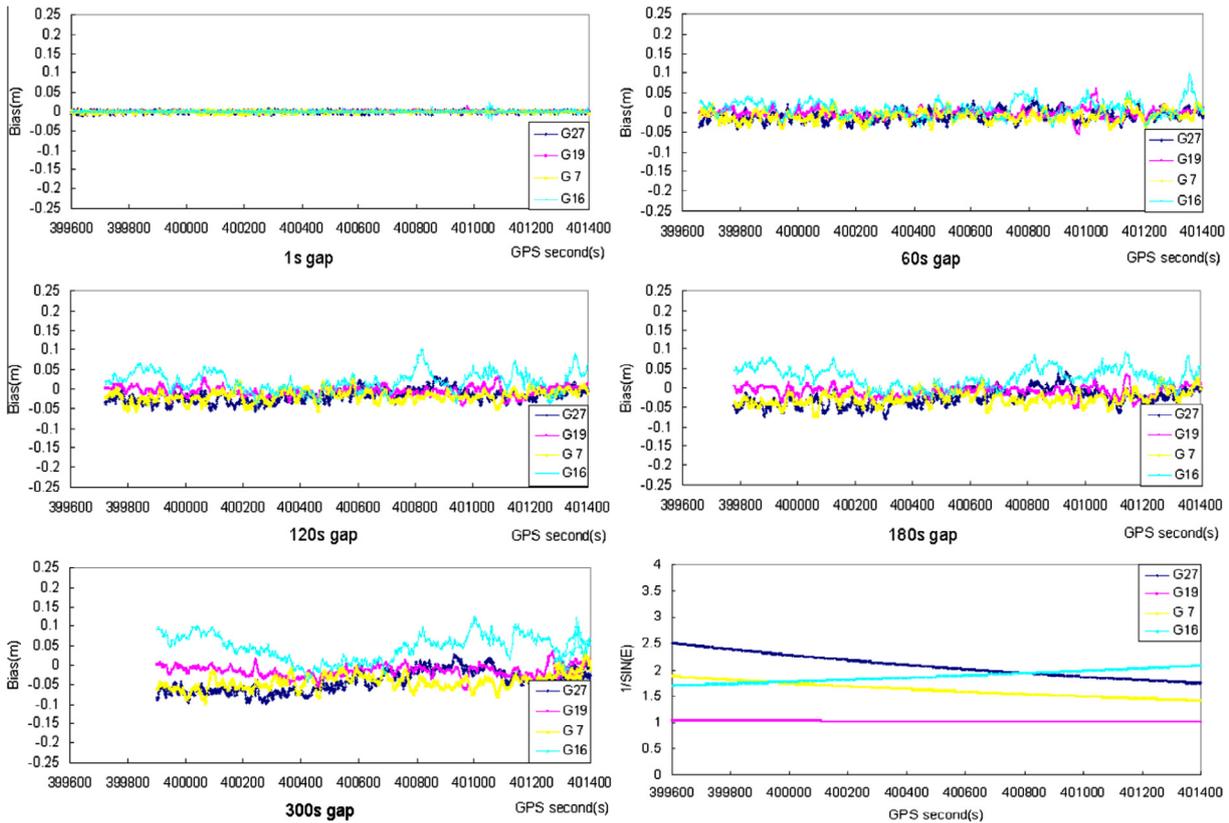


Fig. 3. The predicted error of ionospheric delays at different latencies and the elevation angles of the four satellites.

final positioning results of fixed solution with L3 observations.

The statistical results including the ambiguity success rate and horizontal position accuracy of 24 h (total 86,400 solutions) are summarized in Table 1. Here the success rate is the ratio between the number of successfully ambiguity-fixed solutions and the total number of solutions. The vertical position accuracy is usually two times as large as that of the horizontal one.

Examining Fig. 4 and Table 1, one can see that when the latency of the predicted atmospheric delays is only 1 s, the positioning error of WL fixed solution is small. With longer latency, the lower precision of the WL solution and lower success rate is because the predicted atmospheric errors gradually grow. When the latency is up to 180 s, the positioning accuracy decreases to about 5 cm and the fixing success rate is degraded to about 97%. As the final L3 fixed solution is free of ionospheric delays, the positioning errors

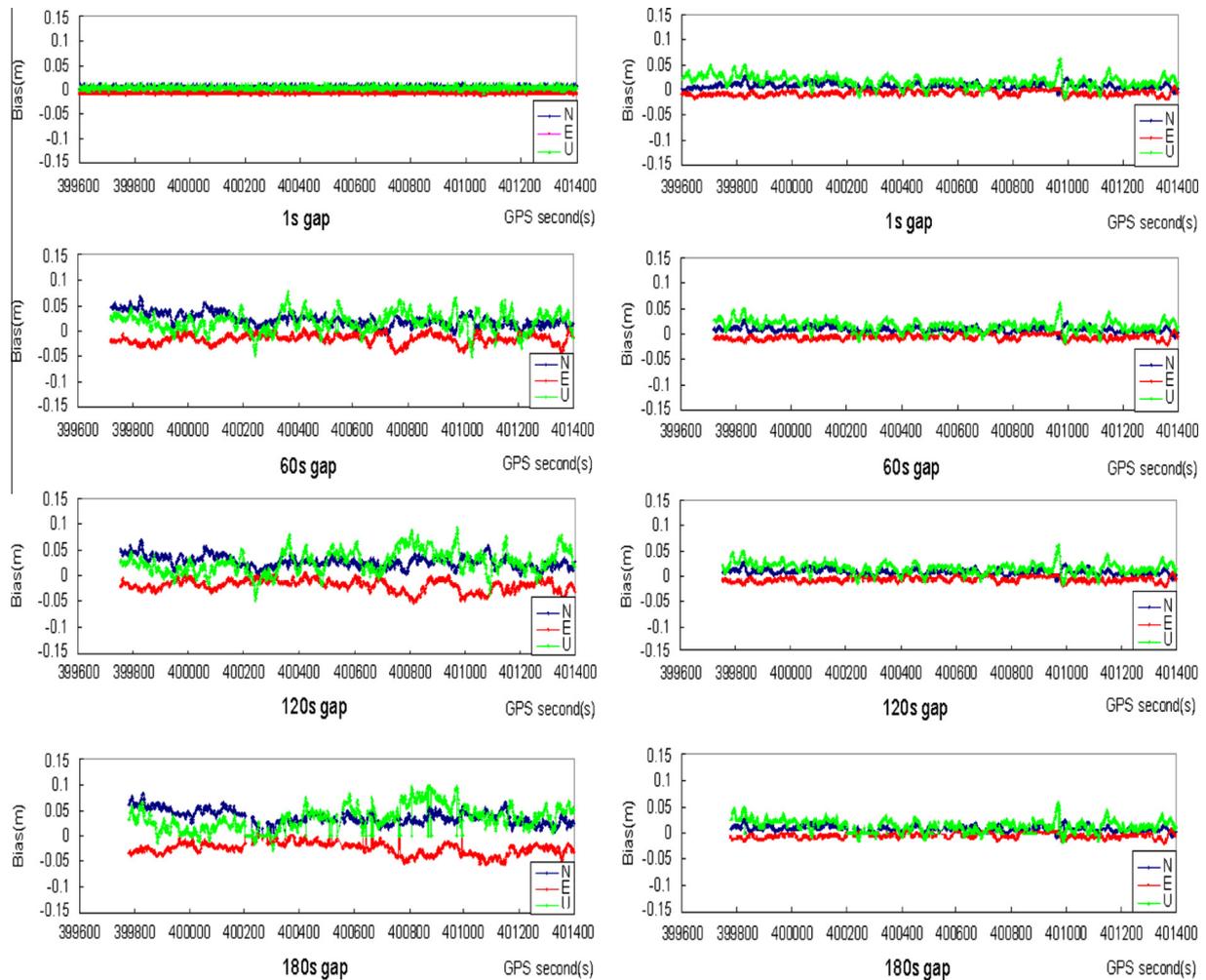


Fig. 4. Positioning error of WL and L3 fixed solution at different latencies.

at different prediction latencies are nearly the same. Therefore, advantage of L3 fixed solution over WL fixed solution becomes more significant with the prediction latency increases. Even if the latency up to 180 s, a success rate about 96% for single-epoch ambiguity resolution can still be achieved with the proposed method in this paper. Moreover, in general, once the WL ambiguity is successfully fixed, the final L3 fixed solution can be achieved with the constraint of high-precision WL solution information. Hence, a way to improve WL fixing success rate will be investigated in future research.

4.2. Kinematic experiment

In this section, we assess this method for different dynamic situations including land walking, ship-borne, airborne and space-borne cases.

4.2.1. Land walking scenario

The data set examined here is a moving trajectory recorded in May, 2008 for approximately 5 h. The 1-Hz GPS data was collected using a Trimble dual-frequency

GPS receiver on the top of a four-story building at School of Geodesy and Geomatics, Wuhan University, China. Generally 6–10 satellites were tracked during the observational period. To simulate signal interruption or loss of lock in practice, all satellite signals are blocked artificially every other hour, i.e. to make all the satellites occur loss of lock simultaneously. Ambiguity-fixed PPP solution was carried out and the positioning errors are presented in Fig. 5. One can find that the signal interruptions cause a long re-initialization period for fixing new ambiguities. During the re-initialization stage, PPP user cannot obtain stable desired accuracy.

We applied the proposed method to process the same test data. At the beginning, the first initialization still requires about 15 min, after which, the integer ambiguities were fixed to their correct numbers. Then the ionospheric and tropospheric delays could be generated and predicted for the subsequent epochs, and a single-epoch ambiguity resolution was applied with the predicted atmospheric delays. The positioning error was shown in Fig. 6. Comparing Fig. 5 with Fig. 6, single-epoch ambiguity resolution with the predicted atmospheric delays removes the

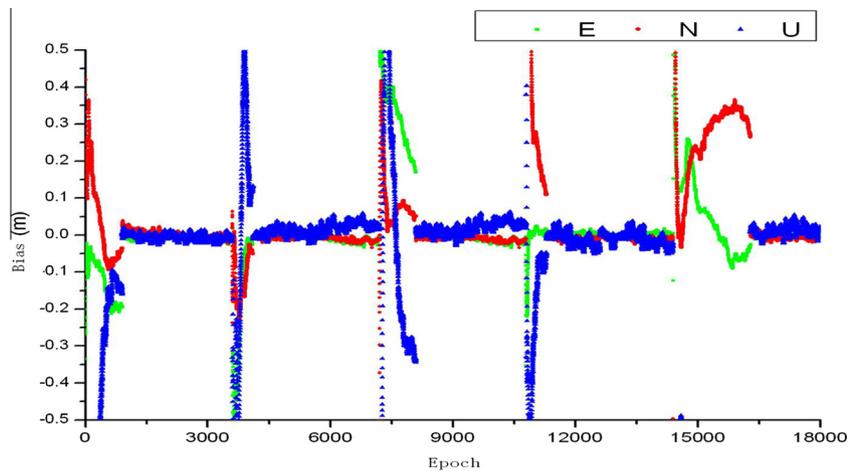


Fig. 5. Positioning errors of a kinematic PPP fixed solution.

re-initialization stage. After the first initialization, all the epochs achieve the desired accuracy continuously. The proposed instantaneous method is robust against cycle slips and data gaps, while still capable of providing cm-level (better than 5 cm horizontally, and within 1 dm vertically) positioning accuracy. This will greatly enhance the applicability of PPP technology, especially real-time PPP in practical engineering.

4.2.2. Ship-borne and airborne data

For the ship-borne experiment, a Leica dual-frequency GPS receiver was set up on a ship. The GPS data sample rate is 1 s. There were tens of signal interruptions throughout the whole session, among which 4–6 satellites (generally 7–8 satellites are visible) lost lock simultaneously for three times. A high-dynamic airborne experiment was also conducted. An Ashtech dual-frequency receiver was installed on an airplane, the sample rate is 1 s. In this experiment, there are two epochs where cycle slips occurred on 4 and 5 satellites respectively.

For both experiments, we performed PPP in a float solution and a fixed solution. The positioning errors in each

case are given in Figs. 7–10. Generally speaking, a PPP solution would usually not be significantly disturbed if occasionally only 1~2 satellites suffer from cycle slips at the same epoch because most of ambiguities have not been reset. However, the solution can evidently be destroyed if loss of lock happens on more than 3~4 phase observations simultaneously (especially if less than 4 satellites are continuously tracked). Examining Figs. 7 and 9, re-convergence will be triggered in PPP float solution when data gap occurs. After applying the proposed rapid re-initialization method, the instantaneous ambiguity resolution could be achieved, so the phase arcs are connected and the PPP solution could be continuous as shown in Figs. 8 and 10.

4.2.3. Space-borne data

This space-borne experiment was designed to demonstrate the feasibility of the proposed method for high dynamic situation. We applied the method to process the GRACE on-board GPS measurement (10 s sampling interval). There are two epochs at which cycle slips were simulated on 5 satellites for both receivers of Grace A and B. The orbit error of the float PPP solution is given in Figs. 11

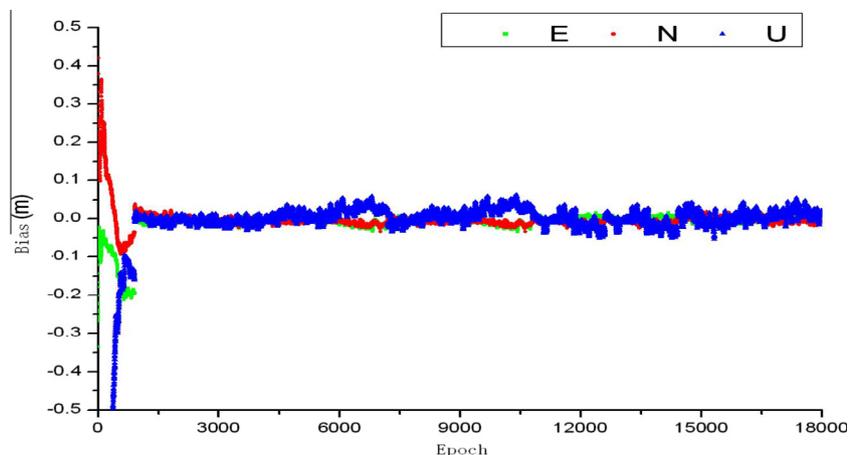


Fig. 6. Positioning errors of kinematic PPP with instantaneous AR.

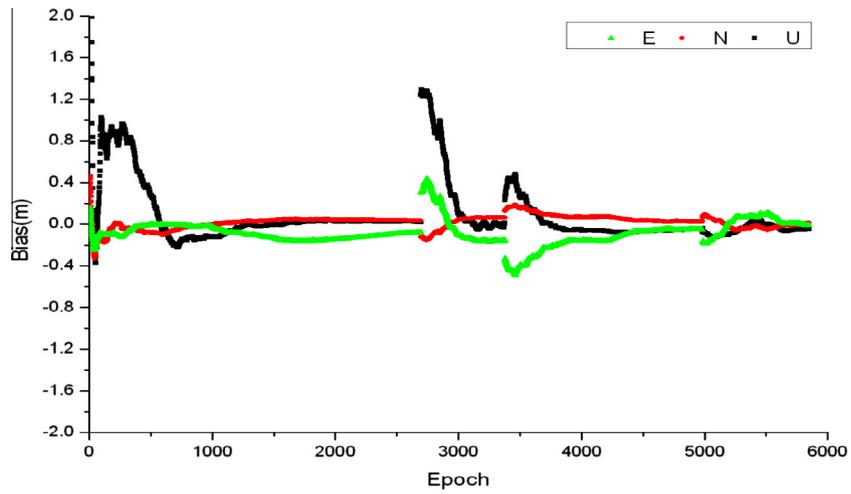


Fig. 7. Ship-Borne PPP with re-initialization.

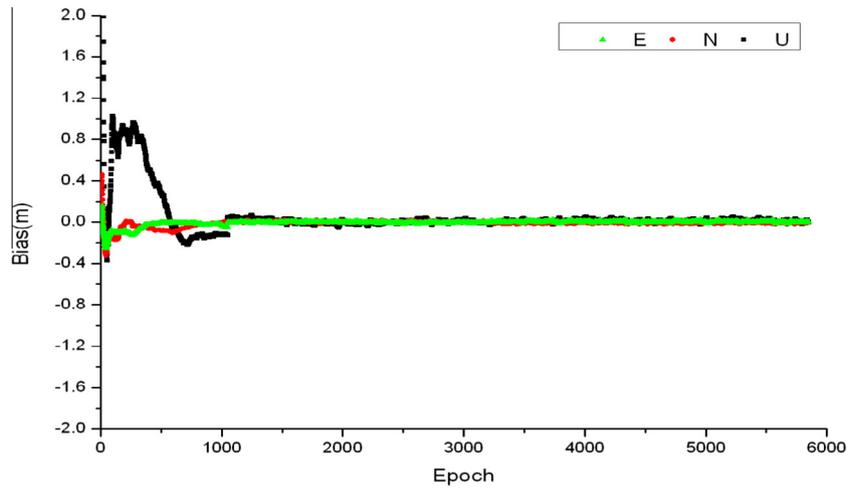


Fig. 8. Ship-borne PPP solution with instantaneous AR.

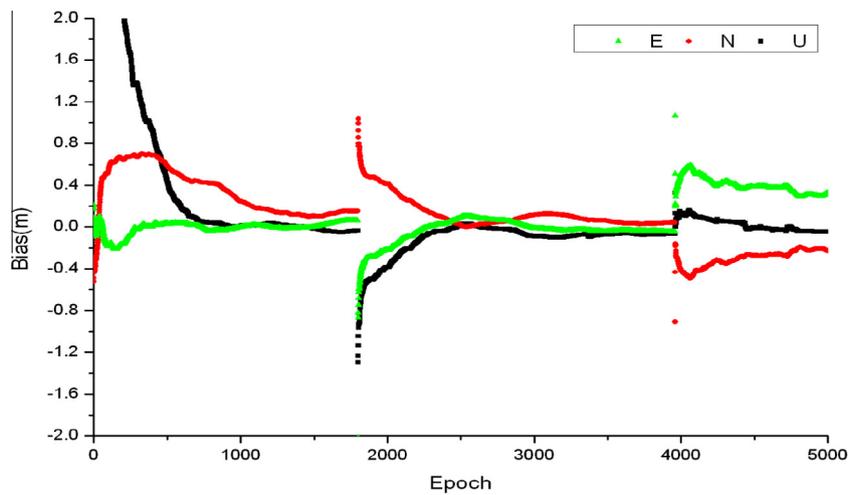


Fig. 9. Airborne PPP with re-initialization.

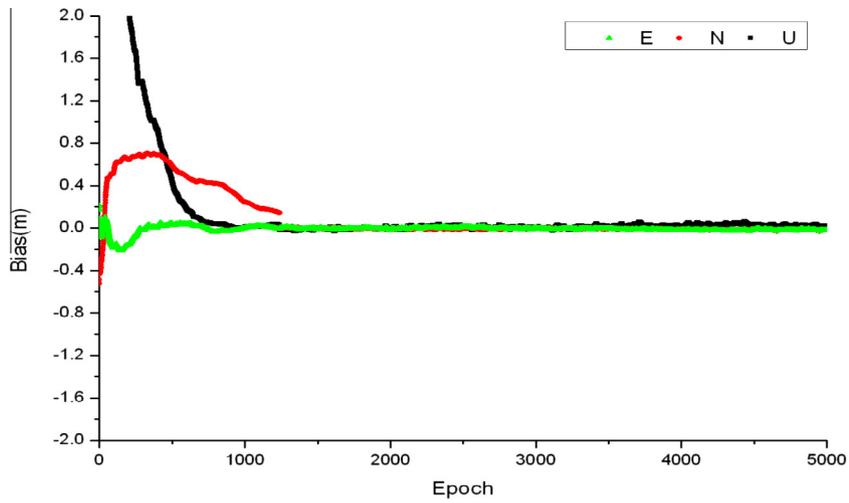


Fig. 10. Airborne PPP solution with instantaneous AR.

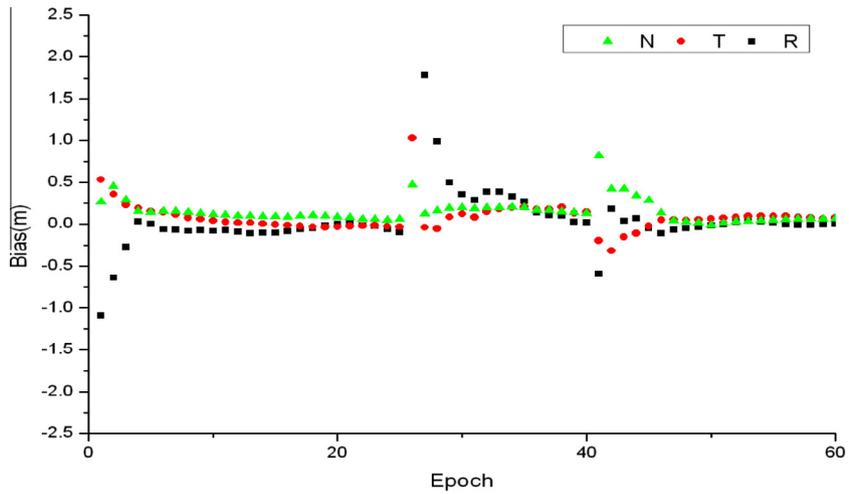


Fig. 11. PPP solution of Grace-A based traditional model.

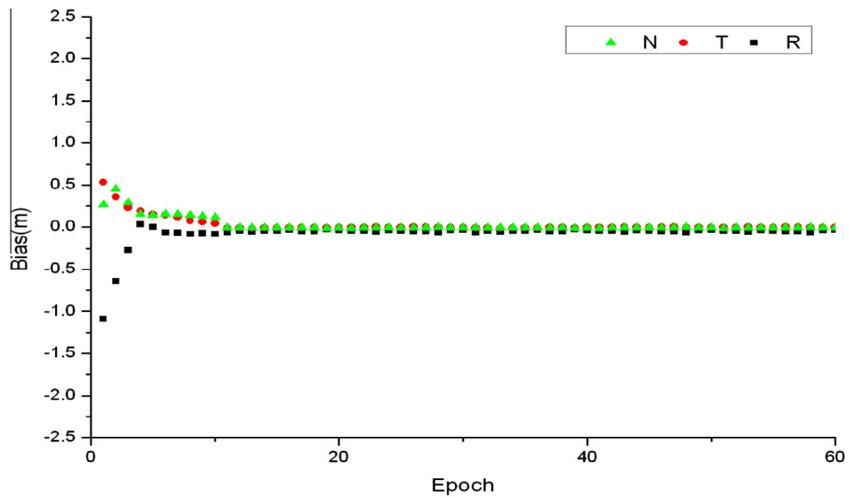


Fig. 12. PPP solution of Grace-A with instantaneous AR.

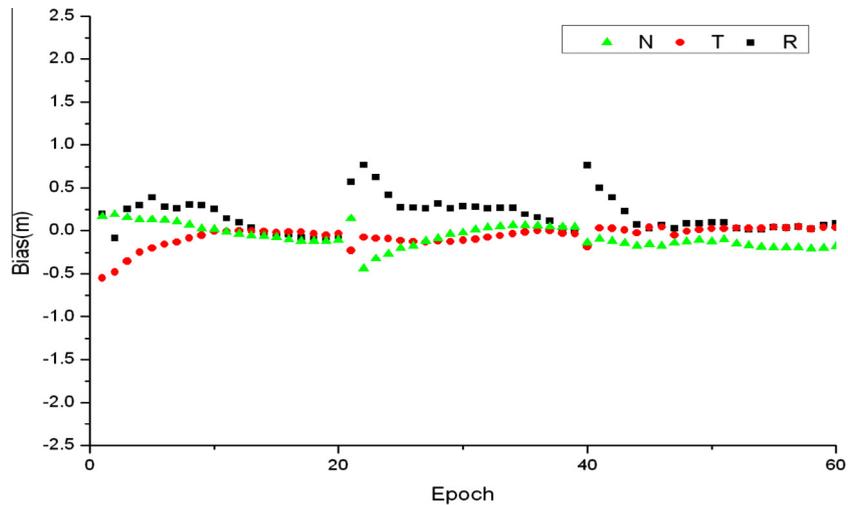


Fig. 13. PPP solution of Grace-B based traditional model.

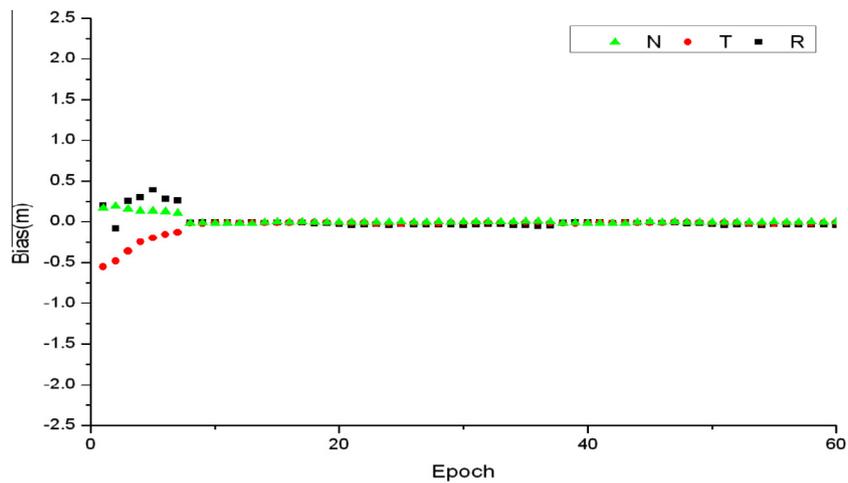


Fig. 14. PPP solution of Grace-B with instantaneous AR.

and 13, and the orbit error of the fixed solution where the instantaneous ambiguity resolution method was applied is shown in Figs. 12 and 14 (JPL orbit is the reference, its interval is 1 min. R , T , N denote radial, tangential and normal direction respectively). One finds that continuous high-accuracy orbit results can be obtained when the instantaneous ambiguity resolution algorithm was applied.

5. Conclusions and discussions

Comprehensive experiments in different observational environments have shown that the proposed method can fix the ZD integer ambiguities successfully with single epoch observations even when all satellites were interrupted. Even when the signal interruption lasts up to 180 s, the instantaneous ambiguity fixing can be achieved by using the proposed method.

The span of data gap could possibly be extended if more precise temporally atmospheric prediction is available or OTF technology is applied. Furthermore, loss of lock does

not always occur on all carrier-phase measurements at one epoch, so the continuous phase measurements on which no cycle slips has occurred can be used to constrain the search space of the ambiguity candidates, which will also contribute to extending the latency of the predicted atmospheric delay. After GPS modernization, especially when multi-frequency (Feng, 2008) as well as multi-system data becomes available, the method proposed in this paper can significantly enhance the applicability of real-time PPP in engineering applications.

However, longer gaps between the GNSS signal interruptions, extreme atmospheric conditions (e.g. ionospheric storm or weather fronts), as well as a more complicated observation environment should be taken into account. In many cases, the number of remaining satellites in view is too low, and after an interruption, satellite availability is recovered gradually. Degraded observability conditions may lead to inaccurate estimations of some of the terms of the phase observation equation, which, in the worst case, could also result in a bad ambiguity resolution. Further

evaluation of this method for more complicated scenarios will be carried out in the subsequent studies.

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